A novel approach to Isoscaling: the role of the order parameter m=(N-Z)/A

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The ratio of the isotope yields, R_{21}, between two similar reaction systems with different N=Z ratios can be expressed by the following isoscaling relation[1,2]:

$$R_{21}(N, Z) = C \exp(\alpha N + \beta Z) \tag{1}$$

where $\alpha=(\mu^1{}_n-\mu^2{}_n)/T$, $\beta=(\mu^1{}_p-\mu^2{}_p)/T$ are isoscaling parameters, representing the differences of the neutron (or proton) chemical potentials between systems 1 and 2, divided by the temperature. C is a constant.

Pursuing the question of phase transitions, in the Laudau approach [3,4] the ratio of the free energy (per particle) to the temperature is written in terms of an expansion:

$$\frac{F}{T} = \frac{1}{2}am^2 + \frac{1}{4}bm^4 + \frac{1}{6}cm^6 - m\frac{H}{T} \tag{2}$$

where m is the order parameter, H is its conjugate variable and a−c are fitting parameters. In our case m = (N-Z)/A.

Because of the symmetries of the free energy when we take the ratio between two different systems, presumably at the same temperature T and density ρ, we can obtain the difference between the free energies by taking the ratio dividing each experimental yield by the ^{12}C yield following in Ref. [5, 6] between two systems as in Eq. (1) as:

$$\frac{-\ln(R_{21}(m))}{A} = \Delta H / Tm + \text{constant}\ , \tag{3}$$

where $\Delta H/T = H_1/T - H_2/T$. Comparing the latter equation with Eq.(1) we obtain: $\Delta H/TmA = \alpha N + \beta Z$ i.e. $\alpha = -\beta = \Delta H/T$.

For an experiment, the projectiles ^{64}Zn, ^{70}Zn and ^{64}Ni,were used to irradiate targets of ^{58}Ni, ^{64}Ni, ^{112}Sn, ^{124}Sn, ^{197}Au and ^{232}Th at 40A MeV. Isotopes were detected inclusively at $\theta = 20^o$ using quad-Si detector telescope. Isotopes are clearly identified up to Z<= 18. The measured energy spectrum of each isotope was integrated using a moving source fit to evaluate the multiplicity.

The scaling of Eq. (3) is satisfied for this set of data as seen in Fig. 1. Compared to ‘traditional’ isoscaling where a fit is performed for each detected charge Z (or each N) we see that all the data collapse into one curve.

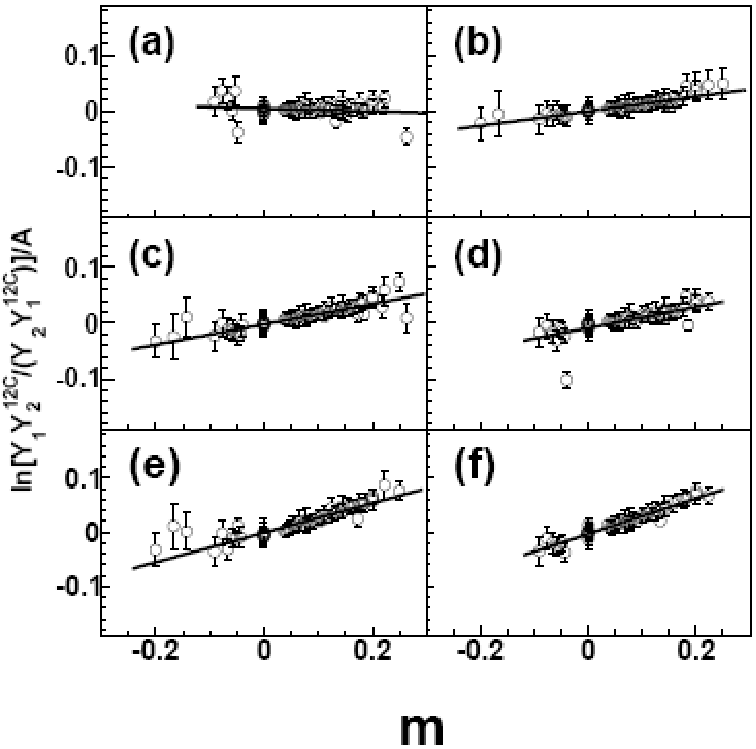

FIG. 1. Experimental ratios vs m for isotopes with 6≤Z≤13 for (a) $\frac{^{64}Ni^{232}Th}{^{70}Zn^{197}Au}$, (b) $\frac{^{64}Ni^{112}Sn}{^{64}Ni^{58}Ni}$, (c) $\frac{^{64}Ni^{124}Sn}{^{64}Ni^{64}Ni}$, (d) $\frac{^{64}Ni^{197}Au}{^{64}Ni^{112}Sn}$, (e) $\frac{^{64}Ni^{124}Sn}{^{70}Zn^{58}Ni}$ and (f) $\frac{^{64}Ni^{124}Sn}{^{70}Zn^{112}Sn}$, respectively at 40MeV/A. The lines are the results of a linear fit according to Eq. (3).

We can further elucidate the role of the external field H/T writing the Landau expansion and 'shifting' the order parameter by m_s which is the position of the minimum of the free energy. Such a position depends on the neutron to proton concentration of the source [4]. Thus

$$\frac{F}{T} = \frac{1}{2}a(m - m_s)^2 + \frac{1}{4}b(m - m_s)^4 + \frac{1}{6}c(m - m_s)^6 . \qquad (4)$$

The Landau approach should be equivalent to it under certain conditions. In our experimental case that b and c are of comparable magnitude to parameter a, we can easily obtain:

$$\frac{\Delta H}{T} mA = \alpha \Delta m_s (N - Z) - \frac{1}{2}a\left(m_{s1}^2 - m_{s2}^2\right)A = \alpha N + \beta Z , \qquad (5)$$

$$\alpha + \beta = -a\left(m_{s1}^2 - m_{s2}^2\right), \qquad (6)$$

Eq. (6) shows that the two approaches are equivalent and that m is an order parameter if $\alpha + \beta = 0$ i.e. neglecting $O(m^2_s)$ terms in the external field. In Fig. 2 we plot $\alpha + \beta$ vs. $m^2_{s1} - m^2_{s2}$, unfortunately the error bars are rather large but we can see a systematic deviation from zero as expected from Eq. (6) for large differences in concentration. This indicates that, at the level of sensitivity so far achieved with data of this type the presence of higher order terms in m is difficult to quantify. Thus, within the error bars, m could be considered an order parameter when relatively neutron (or proton) rich sources are considered. In particular, phase transitions in finite systems could be studied using the same language of macroscopic systems i.e., 'turning on and off' an external field [4].

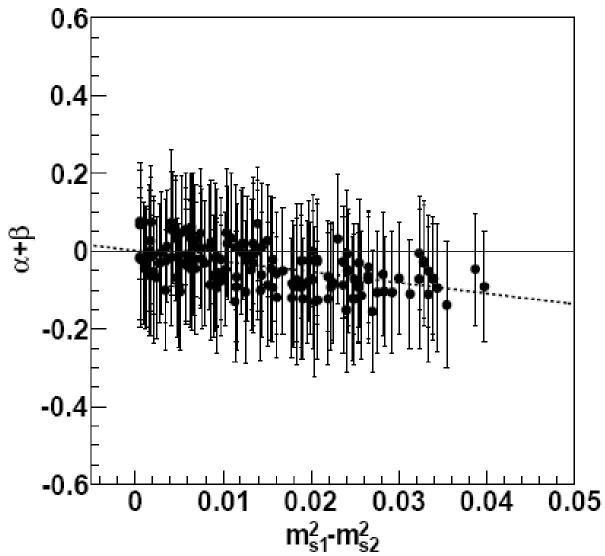

FIG. 2. $\alpha(Z)+\beta(N)$ vs the difference (solid circles) in concentration for the two reaction systems for the case of Z=N=7. The dotted line is the result of a linear fit.

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